It follows from (31b) that

$$C = E = p_1[\psi(q^1) - \psi(q^0)],$$

while the change in expenditure on x is

$$\begin{aligned}\Delta &\equiv p_1[h^1(p, q^0, y) - h^1(p, q^1, y)] \\ &= p_1[\psi(q^1) - \psi(q^0)] + p_1\{\bar{h}^1(p, y + p_1\psi(q^0)) - \bar{h}^1(p, y + p_1\psi(q^1))\} \\ &\lesseqgtr C, E \quad \text{as} \quad \frac{\partial \bar{h}^1}{\partial y} = \frac{\partial h^1}{\partial y} \gtreqless 0. \end{aligned} \tag{32}$$

Thus, if x_1 is a normal good and a perfect substitute for q, the change in the expenditure on x_1 understates the true benefit from an increase in q. In this case, moreover, there are no income effects in the demand curve for q, so that the compensating and equivalent variations coincide. Apart from this special case, however, there does not appear to be any determinate relation between Δ and C or E.

NON-USE VALUES

This above framework can be used to shed some light on the concept of existence value due originally to Krutilla (1967). This is based on the notion that, even if he did not consume any of the x's that are associated with q, an individual might still feel some improvement in q and be willing to pay something to secure it. How can this be explained in terms of the utility model discussed above?

Smith and Desvousges (1986) have made an important distinction between existence values under conditions of certainty and uncertainty. The phenomenon of consumer choices under uncertainty -e.g. the individual does not know whether or not he will want in the future to consume certain x's that are associated with q - raises many important issues that transcend the theory developed above, which is firmly rooted in the context of decisions under certainty. Accordingly, I focus here on the concept of existence values under the conditions of certainty - an individual places some value on an improvement in. q even though he does not himself consume any of the x's that might be associated with q, and has no doubt that he will never consume these goods in the future. Under these circumstances, how can we use the theoretical framework developed above to give some operational meaning to this concept?

Two quantities identified above may have some bearing on this question. The first is based on the decomposition in (11). Suppose that Weak Complementarity does not apply so that $\partial u/\partial q > 0$ even when there is zero consumption of x's that are conventionally associated with q. In that case one could regard the quantity

$$m(\tilde{p}_I, p_{\bar{I}}, q^0, u^0) - m(\tilde{p}_I, p_{\bar{I}}, q^1, u^0) = C - \int_{p_I}^{\tilde{p}_I} \sum_{i \in I} \left[g^i(p, q^1, u^0) - g^i(p, q^0, u^0) \right] dp_i \qquad (33)$$

as a measure of the non-use benefits associated with the improvement in q - these are the benefits that would accrue to the individual even if he were consuming none of the x_I's. Operationally, one would measure them by computing C from the indirect utility function using (3), and then subtracting the area between the compensated demand curves represented by the integral on the RHS of (33). of course, if Weak Complementarity holds, this quantity is zero. As already noted, that would apply to the semi-log demand function (8). Interestingly, it does <u>not</u> apply to another common functional form, the linear ordinary demand function

$$x_1 = h^1(p_1, p_2, q, y) = \alpha - \beta(p_1/p_2) + \gamma(y/p_2) + \delta q. \tag{34}$$

It can be shown that the corresponding compensated demand function $g^1(p,q,u)$ is independent of q so that the integral in (11) and (33) is zero and

$$C = m(\tilde{p}_1, p_2, q^0, u^0) - m(\tilde{p}_1, p_2, q^1, u^0) \tag{35}$$

where the cut-off price is

$$\tilde{p}_1 = \frac{p_2}{\beta}\left(\alpha + \gamma\frac{y}{p_2} + \delta q^1\right). \tag{36}$$

In this case, therefore, <u>all</u> of the benefit from a change in q is associated with term $[m(\tilde{p}, q^0, u^0) \cdot m(\tilde{p}, q^1, u^0)]$. For this reason it may appear unsatisfactory to equate that quantity with the notion of non-use or "existence" value.

The other candidate is the quantity C* (and E*) defined in (19) and (22) in connection with the utility representation (18). That is to say, if the utility function is represented by (18) rather than (17), one could regard the "extra" component of benefits over and above $\bar{C}$ or $\bar{E}$ as a form of non-use value. This interpretation was, indeed, suggested above. An extreme example arises when the utility function takes the form

$$u(x, q) = T[\bar{u}(x), q] \tag{37}$$

i.e. $\bar{u}(x,q)$ is completely independent of q. In that case the demand function for the x's are entirely independent of q - $x_i = \bar{h}^i(p, y)$ all i — but the individual still places some value on changes in q. From (19)

$$C = c' \tag{38}$$

where C* is defined by (20). In this special case the revealed preference approach provides no information (except to confirm that $\bar{C} = \bar{E} = 0$) and the only way to measure C is through some form of contingent valuation or contingent behavior experiment. If the utility function has the general form (18), but not the extreme form in (37), a similar conclusion would apply: the only way to measure the non-use benefits C* and E* is by contingent valuation and/or contingent behavior procedures.

REFERENCES

W. Michael Hanemann (1980a) "Measuring the Worth of Natural Resource facilities: Comment" Land Economics Vol. 56 November 1980 pp. 482-486

w. Michael Hanemann (1980b) "Quality Changes, Consumer's Surplus and Hedonic Price Indices, " University of California, Department of Agricultural and Resource Economics, Working paper No- 116, Berkeley, November 1980.

w. Michael Hanemann (1981) "Some Further Results on Exact Consumer's Surplus" University of California, Department of Agricultural and Resource Economics, Working paper No. 190, Berkeley, December, 1981.

w. Michael Hanemann (1985) "Some Issues in Discrete - and Continuous - Response Contingent Valuation Studies, " Northeastern Journal of Agricultural Economics, April 1985

John V. Krutilla (1967) "Conservation Reconsidered, " American Economic Review Vol. 57, September 1967, PP 777-786

Karl-Goran Maler (1971), "A Method of Estimating Social Benefits From Pollution Control, " Swedish Journal of Economics vol. 73, No. 1, pp 121-133

Karl-Goran Maler (1974), Environmental Economics: A Theoretical Inquiry (Baltimore: The Johns Hopkins University Press)

v. Kerry Smith and William H. Desvousges (1986), Measuring Water Quality Benefits (Boston: Kluwer-Nijhoff Publishing Co.)

APPENDIX

WILLINGNESS TO PAY AND WILLINGNESS TO ACCEPT: HOW MUCH CAN THEY DIFFER?

Consider an improvement in the exogenous variables comprising an individual's choice set. Two possible monetary measures of the gain in her welfare are the compensating variation (C) and the equivalent variation (E). In the present context, these correspond, respectively, to the maximum amount the individual would be willing to pay (WTP) to secure the change and the minimum compensation that she would be willing to accept (WTA) to forego the change. Howmuch can the two differ, and what are the factors that determine the difference? These questions were addressed by Robert Willig (1976) in his pathbreaking paper on the welfare measurement of price changes. Willig argued that C and E are likely in practice to be fairly close in value, and he showed that the difference depends directly on the size of the income elasticity of demand for the commodity whose price changes.

In many empirical studies, however, analysts seek to obtain money measures of welfare changes due not to price changes but to changes in the availability of public goods or amenities, changes in the qualities of commodities, or changes in the fixed quantities of rationed goods. Karl-Göran Mäler (1974) was perhaps the first to show that the concepts of C and E can readily be extended from conventional price changes to quantity changes such as these. Subsequently, Alan Randall and John Stoll (1980) examined the duality theory associated with fixed quantities in the utility function and showed that, with appropriate modifications, Willig's formulas for bounds on C and E do, indeed, carry over to this setting. Within the environmental literature and elsewhere, Randall and Stoll's results have been widely interpreted as implying that WTP and WTA for changes in environmental-amenities should not differ greatly unless there are unusual income effects.[1] However, recent empirical work using various types of interview procedures has produced some evidence of large disparities between WTP and WTA measures--for example, Richard C. Bishop and Thomas A. Hebertein (1979) and several studies described by Irene M. Gordon and Jack L. Knetsch (1979), and by Knetsch and Sinden (1984). This has led to something of an impasse: How can the empirical evidence of significant differences between WTP and WTA be reconciled with the theoretical analysis suggesting that such differences are unlikely? Can they be explained entirely by unusual income effects or by peculiarities of the interview process?

In this note I reexamine Randall and Stoll's analysis and show that, while it is indeed accurate, its implications have been misunderstood. For quantity changes there is no presumption that WTP and WTA must be close in value and, unlike price changes, the difference between WTP and WTA depends not only on an income effect but also on a substitution effect. By the latter, I mean the ease with which other privately marketed commodities can be substituted for the given public good or fixed commodity, while maintaining the individual at a constant level of utility. I show that, holding income effects constant, the smaller the substitution effect (i.e., the fewer substitutes available for the public good) the greater the disparity between WTP and WTA. This surely coincides with common intuition. If there are private goods which are readily substitutable for the public good, there ought to be little difference between an individual's WTP and WTA for a change in the public good. But, if the public good has almost no substitutes (e.g., Yosemite National Park or, in a different context, your own life), there is no reason why WTP and WTA could not differ vastly--in the limit, WTP could equal the individual's entire (finite) income while WTA could be infinite. My argument is developed in the following two sections. Section I deals specifically with the two polar cases of perfect substitution and zero substitution between the public good and available private goods. Section II deals with Randall and Stoll's extension of Willig's formulas and shows that their bounds are, in fact, consistent with substantial divergences between WTP and WTA.

I. Two Polar Cases

The theoretical setup is as follows. An individual has preferences for various conventional market commodities whose consumption is denoted by the vector x as well as for another commodity whose consumption is denoted by q.[2] This could represent the supply of a public good or amenity; it could be an index of the quality of one of the private goods; or it could be a private commodity whose consumption is fixed by a public agency.[3] The key point is that the individual's consumption of q is fixed exogenously, while she can freely vary her consumption of the x's. These preferences are represented by a utility function, $u(x, q)$, which is continuous and nondecreasing in its arguments (I assume that the x's and q are all "goods") and strictly quasiconcave in x. The individual chooses her consumption by solving

$$(1)\qquad \max_{x} u(x, q) \text{ subject to } \Sigma p_i x_i = y$$

taking the level of q as given. This yields a set of ordinary demand functions, $x_i = h^i(p, q, y)$, $i=1, \ldots, N$, and an indirect utility function, $v(p, q, y) \equiv u[h(p, q, y), q]$, which has the conventional properties with respect to the price and income arguments and also is increasing in q.[4] Now suppose that q rises from q^0 to $q^1 > q^0$ while prices and income remain constant at (P, y). Accordingly, the individual's utility changes from $u^0 \equiv V(p, q^0, y)$ to $u^1 \equiv V(p, q^1, y) \geq u^0$. Following Mäler, the compensating and equivalent variation measures of this change are defined, respectively, by[5]

$$V(p, q^1, y - C) = V(p, q^0, y) \tag{2}$$

$$V(p, q^1, y) = V(p, q^0, y + E). \tag{3}$$

Dual to the utility maximization in (1) is an expenditure minimization: Minimize $\Sigma p_i x_i$ with respect to x subject to $u = u(x, q)$, which yields a set of compensated demand functions, $x_i = g^i(p, q, u)$, $i = 1, \ldots, N$, and an expenditure function, $m(p, q, u) \equiv \Sigma p_i g^i(p, q, u)$, which has the conventional properties with respect to (p, u) and is decreasing in q. In terms of this function, C and E are given by

$$C = m(p, q^0, u^0) - m(p, q^1, u^0) \tag{2'}$$

$$E = m(p, q^0, u^1) - m(p, q^1, u^1). \tag{3'}$$

It is evident from (2) and (3) that $0 \leq C \leq y$ while $E \geq 0$.[6] The questions at issue are: (1) Is it true that $E/C \approx 1$? (2) What factors affect this ratio? As a first cut at an answer, I compare two polar cases. In the first case at least one private good--say, the first--is a perfect substitute for some transformation of q. Thus, the direct utility function assumes the special form

$$U(x, q) = \bar{u}[x_1 + \psi(q), x_2, \ldots, x_N] \tag{4}$$

where $\psi(\bullet)$ is an increasing function and $\bar{u}(\cdot)$ is a continuous, increasing, strictly quasiconcave function of N variables. As W. M. German (1976) has shown, the resulting indirect utility function is

$$(5) \qquad v(p, q, y) = \bar{v}[p_1, p_2, \ldots, p_N, y + p_1 \psi(q)]$$

where $\bar{v}(\cdot)$ is the indirect utility function corresponding to $\bar{u}(\cdot)$. Substitution of (5) into (2) and (3) yields the following:[7]

PROPOSITION 1: If at least one private market good is a perfect substitute for q, then $C = E$.

At the opposite extreme, I assume that there is a zero elasticity of substitution not just between q and x_1 but between q and <u>all</u> the x's. Thus, the direct utility function becomes

$$(6) \qquad u(x, q) = \bar{u}\left[\min\left(q, \frac{x_1}{\alpha_1}\right), \ldots, \min\left(q, \cdot\right)\right]$$

where $\alpha_1, \ldots, \alpha_N$ are positive constants and $\bar{u}(\cdot)$ is a conventional direct utility function. In this case the indirect utility function $v(p, q, y)$ has a rather complex structure and changes its form in different segments of (p, q, y) space. It will be sufficient for my purposes to focus on just one of these segments. Suppose that $q \leq y/\Sigma p_i \alpha_i$; then the maximization of (6), subject to the budget constraint, yields ordinary demand functions and an indirect utility function of the form $x_i = h^i(p, q, y) = \alpha_i q$, and $u = v(p, q, y) = \bar{u}(q, \ldots, q) \equiv w(q)$. In this region of (p, q, y) space, the individual does not exhaust her budget, and her marginal utility of income is therefore zero. Now suppose that $q^0 \leq y/\Sigma p_i \alpha_i$ and $q^1 > q^0$. Since $v(p, q^1, y) > w(q^0)$, it is evident from (2) that the individual would be willing to pay some positive but limited amount C to secure this change. However, for any positive quantity E, no matter how large, $V(p, q^0, y + E) = v(p, q^0, y) = w(q^0)$. This implies the following:

PROPOSITION 2: If there is zero substitutability between q and each of the private market goods, it can happen that, while the individual would only be willing to pay a finite amount for an increase in q, there is no finite compensation that she would accept to forego this increase.

It should be emphasized that this result obtains only in a portion of (p, q, y) space; in other regions, even with (6), E would be finite.[8] However, the result in Proposition 2 can also be established for other utility functions that permit some substitutability between q and the x's as long as the indifference curves between q and each of the x's become parallel to the q axis at some point. The lesson to be learned from these two propositions is that the degree of substitutability between q and private market goods significantly affects the relation between C and E. In the next section, I show how this observation can be reconciled with the bounds on C and E derived by Randall and Stoll.

II. Randall and Stoll's Bounds

In Order to extend Willig's bounds from price to commodity space, Randall and Stoll focus on a set of demand functions different from those considered above. Suppose that the individual could purchase q in a market at some given price, π. It must be emphasized that this market is entirely hypothetical since q is actually a public good. Instead of (1), she would now solve[9]

$$\max_{x,q} u(x, q) \text{ subject to } \Sigma p_i x_i + \pi q = y. \tag{7}$$

Denote the resulting ordinary demand functions by $x_i = \hat{h}^i(p, \pi, y)$, i = 1, ..., N and $q = \hat{h}^q(p, \pi, y)$. The corresponding indirect utility function is $\hat{v}(p, \pi, y) \equiv u[\hat{h}(p, \pi, y), \hat{h}^q(p, \pi, y)]$. The dual to (7) is: Minimize $\Sigma p_i x_i + \pi q$ with respect to x and q subject to $u = u(x, q)$. This generates a set of compensated demand functions, $x_i = \hat{g}^i(p, \pi, u)$, i = 1, ... N and $q = \hat{g}^q(p, \pi, u)$, and an expenditure function, $\hat{m}(p, \pi, u) \equiv \Sigma p_i \hat{g}^i(p, \pi, u) + \pi \hat{g}^q(p, \pi, u)$. These functions are hypothetical since q is really exogenous to the individual, but they are of theoretical interest because they shed light on the relation between C and E.

For any given values of q, p, and u, the equation,

$$q = \hat{g}^q(p, \pi, u), \tag{8}$$

may be solved to obtain $\pi = \hat{\pi}(p, q, u)$, the inverse compensated demand (i.e., willingness to pay) function for q: $\hat{\pi}(\cdot)$ is the price that would induce the individual to purchase q units of the public good in order to attain a utility level of u, given that she could buy private goods at prices p. Let $\pi^0 \equiv \hat{\pi}(p, q^0, u^0)$ and $\pi^1 \equiv \hat{\pi}(p, q^1, u^1)$ denote the prices that would have supported q^0 and q^1, respectively. The two expenditure functions dual to (1) and (7) are related by:

$$m(p, q, u) \equiv \hat{m}[p, \hat{\pi}(p, q, u), u] - \hat{\pi}(p, q, u) \cdot q. \tag{9}$$

This implies that[10]

$$(10) \qquad m_q(p, q, u) = -\hat{\pi}(p, q, u).$$

Combining (10) with (2′) and (3′) yields these alternative formulas for C and E expressed in terms of the willingness-to-pay function:

$$(2'') \qquad C = \int_{q^0}^{q^1} \hat{\pi}(p, q, u^0)\, dq$$

$$(3'') \qquad E = \int_{q^0}^{q^1} \hat{\pi}(p, q, u^1)\, dq.$$

It can be shown that sign $(\hat{\pi}_u)$ = sign $(\hat{h}^q_y)$. Therefore, for given (π, q), the graph of $\hat{\pi}(p, q, u^1)$ lies above (below) that of $\hat{\pi}(p, q, u^0)$, and E > (<) C, accordingly as q is a normal (inferior) good. Figure 1 shows E and C for the case where q is normal: E corresponds to the area $q^0 \alpha \gamma q^1$ while C corresponds to the area $q^0 \beta \delta q$?

Using the technique pioneered by Willig, Randall and Stoll establish bounds on the difference between each of C and E and the area under an inverse ordinary demand function for q. From this, they derive bounds on the difference between C and E. However, the requisite inverse ordinary demand function is obtained in a rather special manner. Given any level of q, we can ask what market price π would induce the individual to purchase that amount of public good if it were available in a market, while still allowing her to purchase the quantity of the x's that she actually did buy at market prices p with income y. In conducting this thought experiment, one needs to supplement her

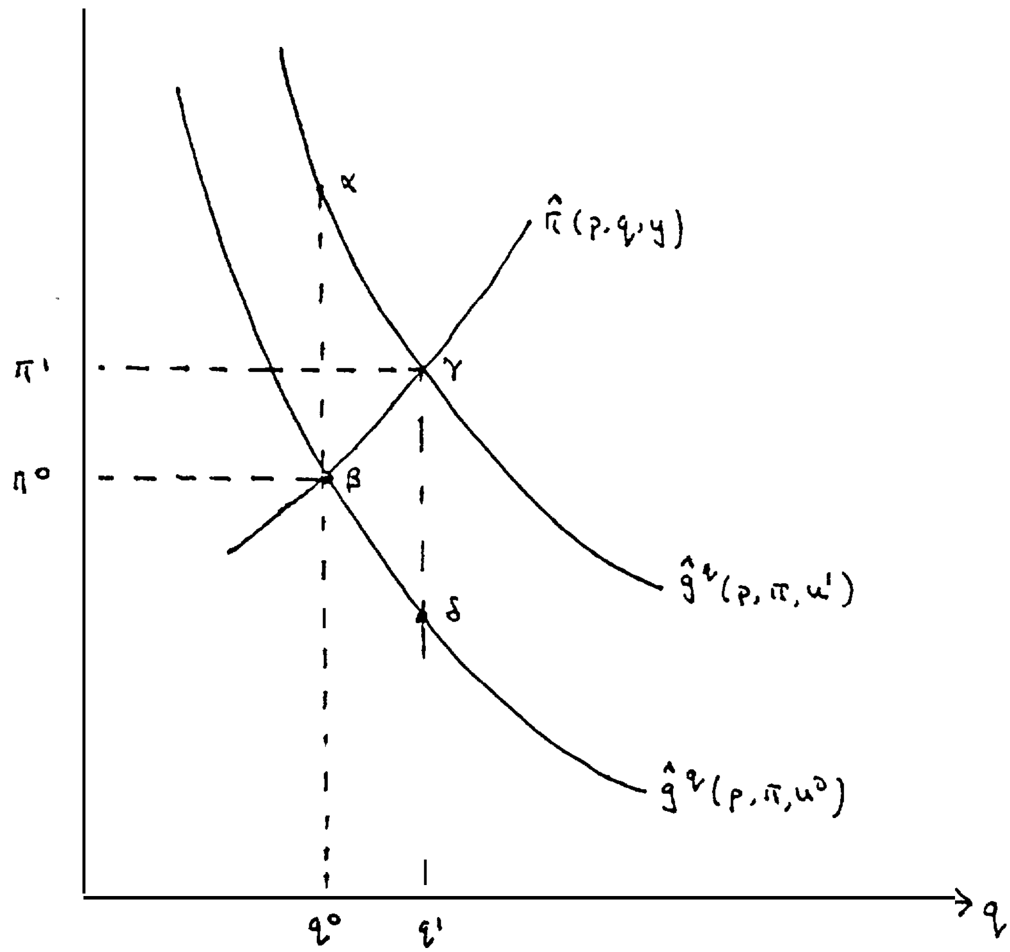

FIGURE 1. WTP and WTA for a Change in q.

income so that she can afford q as well as the x's. Thus, for given (p, q, y), we seek the price π that satisfies

$$q = h^q(p, \pi, y + \pi q). \tag{11}$$

The solution will be denoted by $\pi = \hat{\pi}(p, q, y)$. This inverse function is related to the inverse compensated demand function by the identities[11]

$$\hat{\pi}(p, q, y) \equiv \pi[p, q, v(p, q, y)] \tag{12a}$$

$$\hat{\pi}(p, q, u) \equiv \pi[p, q, m(p, q, u)]. \tag{12b}$$

It follows from (12a) that $\pi^0 \equiv \hat{\pi}(p, q^0, u^0) = \hat{\pi}(p, q^0, y)$ and $\pi^1 \equiv \hat{\pi}(p, q^1, u^1) = \hat{\pi}(p, q^1, y)$--i.e., the graph of $\hat{\pi}(p, q, y)$ as a function of q intersects the graph of $\hat{\pi}(p, q, u^0)$ at $q = q^0$, and the graph of $\hat{\pi}(p, q, u^1)$ at $q = q^1$. This is depicted in Figure 1.[12]

Using the inverse demand function $\hat{\pi}(p, q, y)$, define the quantity

$$A \equiv \int_{q^0}^{q^1} \hat{\pi}(p, q, y)\, dq \tag{13}$$

which corresponds to the area $q^0 \beta \gamma \delta q^1$ in Figure 1. This is a sort of Marshallian consumer's surplus, which is to be compared with C and E. Let

$$\xi = \frac{\partial \ln \hat{\pi}(p, q, y)}{\partial \ln y} \tag{14}$$

be the income elasticity of $\hat{\pi}(p, q, y)$; Randall and Stoll call this the "price flexibility of income." Assume that, over the range from (p, q^0, y) to (p, q^1, y), this elasticity is bounded from below by ξ^L and from above by

ξ^U with neither bound equal to 1. Using the mean-value theorem, as in Willig's equation (18), and the above equations (2′), (3′), (10), (12b), (13), and (14), yields Randall and Stoll's result--namely,

PROPOSITION 3: Assume $\xi^L \leq \xi \leq \xi^U$ where $\xi^L \neq 1$ and $\xi^U \neq 1$. Then,

(i) $$0 \leq \left[1 + (1 - \xi^L)\frac{A}{y}\right]^{\frac{1}{1-\xi^L}} - 1 \leq \frac{E}{y}$$

(ii) $$0 \leq 1 - \left[1 - (1 - \xi^U)\frac{A}{y}\right]^{\frac{1}{1-\xi^U}} \leq \frac{C}{y} \leq 1$$

(iii) If $\xi^U < 1$, or if $\xi^U > 1$ and $1 + (1 - \xi^U)\frac{A}{y} \geq 0$, $\frac{E}{y} \leq \left[1 + (1 - \xi^U)\frac{A}{y}\right]^{\frac{1}{1-\xi^U}} - 1$

(iv) If $\xi^L > 1$, or if $\xi^L < 1$ and $1 - (1 - \xi^L)\frac{A}{y} \geq 0$, $\frac{C}{y} \leq 1 - \left[1 - (1 - \xi^L)\frac{A}{y}\right]^{\frac{1}{1-\xi^L}}$. ∎

Applying a Taylor approximation, as in Willig, and assuming that the conditions in (iii) and (iv) are satisfied, one obtains

(15) $$\xi^L \frac{A^2}{y} \leq E - C \leq \xi^U \frac{A^2}{y}.$$

This is commonly interpreted as implying that C and E are close in value, but whether or not that is correct clearly depends on the magnitudes of (A/y) and the bounds ξ^L and ξ^U. The magnitude of (A/Y) depends in part on the size of the change from q^0 to q? But what can be said about the likely magnitude of the income elasticity, ξ--could it happen, for example, that $\xi^L = \infty$? To answer that question, differentiate (11) implicity

$$\text{(16)} \qquad \frac{\partial\pi(p, q, y)}{\partial y} = - \frac{\hat{h}^q_y(p, \pi, y + \pi q)}{\hat{h}^q_\pi(p, \pi, y + \pi q) + q\hat{h}^q_y(p, \pi, y + \pi q)} .$$

By the Hicks-Slutsky decomposition, the denominator is equal to the own-price derivative of the compensated demand function for q and is nonpositive

$$\hat{g}^q_\pi[p, \pi, v(p, q, y)] = \hat{h}^q_\pi(p, \pi, y + \pi q) + q\, \hat{h}^q_y(p, \pi, y + \pi q) \leq 0.$$

Converted to elasticity form, (16) becomes

$$\text{(16}'\text{)} \qquad \xi = - \frac{\eta(1 - \alpha)}{\varepsilon}$$

where $\eta \equiv (y + \pi q)\,\hat{h}^q_y(p, \pi, y + q\pi)/q$ is the income elasticity of the direct ordinary demand function for q, $\alpha \equiv q\pi/(y + q\pi)$ is the budget share of q in relation to "adjusted" income, and $\varepsilon \equiv \pi\hat{g}^q_\pi[p, \pi, v(p, q, y)]/q$ is the own-price elasticity of the compensated demand function for q. The last term can be related to the overall elasticity of substitution between q and the private market goods $x_1, \ldots, x_N$. By adapting W. E. Diewert's (1974) analysis, it can be shown that, if the prices $p_1, \ldots, p_N$ vary in strict proportion (i.e., $p_i = \theta\bar{p}_i$ for some fixed vector $\bar{p}$), the aggregate Allen-Uzawa elasticity of substitution between q and the Hicksian composite commodity $x_0 \equiv \Sigma\bar{p}_i x_i$, denoted σ_0, is related to the compensated own-price elasticity for q by the formula: $\varepsilon = -\sigma_0(1 - \alpha)$. Hence, (16′) may be written

(16″) $$\xi = \frac{\eta}{\sigma_0}$$

where $\sigma_0 \geq 0$.

This provides an explanation of the results in the previous section. For changes in q , unlike changes in p, the extent of the difference between C and E depends not only on income effects (i.e. , η) but also on substitution effects (i.e., 0.). If, over the relevant range, either $\eta = 0$ (no income effects) or $\sigma_0 = \infty$ (perfect substitution betweenq and one or more of the x's), then $\xi^L = \xi^U = 0$ and, from Proposition 3, C = A = E. On the other hand, if the demand function for q is highly income elastic, or there are very few substitutes for q among the x's so that σ_0 is close to zero, this could generate very large values of ξ and substantial divergences between C and E. Suppose, for example, that, over the relevant range, a lower bound on the income elasticity of $\hat{\pi}(\cdot)$ is $\xi^L = 20$ (e.g., $\eta = 2$ and $\sigma_0 = 0.1$) and $A/y = 0.05$. Then, from Proposition 3 (i and iv), $C/y \leq 0.0345$ while $0.1708 < E/y$, so that E is at least five times larger than C.[13] Higher values of ξ^L would imply even greater differences between C and E.

III. Conclusion

A recent assessment of the state of the art of public good valuation concludes "Received theory establishes that . . . WTP . . . should approximately equal . . . WTA. . . . In contrast with theoretical axioms which predict small differences between WTP and WTA, results from contingent valuation method applications wherein such measures are derived almost always demonstrate large differences between average WTP and WTA. To date, researchers have been unable to explain in any definitive way the persistently observed differences between WTP and WTA measures" (Cummings, Brookshire, and Schulze, p. 41). This paper offers an explanation by showing that the theoretical presumption of approximate equality between WTP and WTA is misconceived. This is because, for public goods, the relation between the two welfare measures depends on a substitution effect as well as an income effect. Given that the substitution elasticity appears in the denominator of (16") and the Engel aggregation condition places some limit on the plausible magnitude of the numerator, this suggests that the substitution effects are likely to exert far greater leverage, in practice, on the relation between WTP and WTA than the income effects. Thus, large empirical divergences between WTP and WTA may be indicative not of some failure in the survey methodology but of a general perception on the part of the individuals surveyed that the private market goods available in their choice set are, collectively, a rather imperfect substitute for the public good under consideration.

FOOTNOTES

[1]This view is expressed by, for example, Myrick Freeman (1979, p. 3); Mark A. Thayer (1981, p. 30); Jack L. Knetsch and J. A. Sinden (1984, p. 508); and Don L. Coursey, William D. Schulze, and John J. Hovis (1984, p. 2).

[2]I am treating q as a scalar here, but it could be a vector without seriously affecting the analysis in this section. In the next section, however, the analysis would become significantly more complex if q were a vector and more than one element of q changed.

[3]These alternative interpretations are offered, respectively, by Mäler, W. Michael Hanemann (1982), and Randall and Stoll.

[4]These properties are established in my earlier paper.

[5]I have taken the liberty of defining C and E as the negative of quantities appearing in Willig and in Randall and Stoll, so that sign (C) = sign (E) = sign $(u^1 - u^0)$.

[6]I assume throughout that $q^1 > q^0$ and $u^1 > u^0$. The analysis could be repeated for a case in which quality decreases and $u^1 < u^0$. In that case, C and E are both nonpositive and correspond, respectively, to the compensation that the individual would be willing to accept to consent to the change and the amount that she would be willing to pay to avoid the change. This would reverse the inequalities presented below, but it would not affect the substance of my argument.

[7]This result carries over, of course, if more than one private good is a perfect substitute for q. In the most general case, $u(x, q) = \bar{u}[x_1 + \psi_1(q), \ldots, x_N + \psi_N(q)]$ and $C = E = \Sigma p_i[\psi_i(q^1) - \psi_i(q^0)]$.

[8]Indeed, if $\bar{h}^i(\alpha_1 p_1, \ldots, \alpha_N p_N, y) \leq q^0$, $i = 1, \ldots, N$, it can be shown that $v(p, q^0, y) = v(p, q^1, y) = \bar{v}(\alpha_1 p_1, \ldots, \alpha_N p_N, y)$ and $C = E = 0$, where $\bar{h}^i(\cdot)$ and $\bar{v}(\cdot)$ are the ordinary demand functions and indirect utility function associated with $\bar{u}(\cdot)$.

[9]It is now necessary to assume that $u(\cdot)$ is strictly quasiconcave in both x and q.

[10]Using subscripts to denote derivatives, differentiate (9) and note that $q = \hat{g}^q(p, \pi, U) = \hat{m}_\pi(p, \pi, U)$ by Shephard's Lemma. Equations similar to (9) through (12) are presented by J. P. Neary and K. W. S. Roberts (1980).

[11]Note that $\hat{\pi}(p, q, y)$ is not an inverse ordinary demand function in the sense of Ronald W. Anderson (1980) because it involves an income adjustment as well as a price effect.

[12]It is commonly supposed that $\pi^0 > \pi^1$ when $q^0 < q^1$--see, for example, Figure 7.12 in Richard E. Just, Darrell L. Hueth, and Andrew Schmitz (1982)--but this is not correct. It can be shown that $\pi^0 \gtreqless \pi^1$ according as $\eta \lesseqgtr (1/\alpha)$. Since $\Sigma\alpha_i\eta_i + \alpha\eta = 1$ by the Engel aggregation condition, where $\alpha_i \equiv p_i x_i/(y + \pi q)$ and $\eta_i \equiv (y + \pi q)\, h^i_y/x_i$, $\pi^0 \lesseqgtr \pi^1$ if and only if $\Sigma\alpha_i \eta_i \lesseqgtr 0$.

[13]This is actually the order of magnitude by which WTA measures exceed WTP measures in the empirical studies summarized in Table 3.2 of Ronald G. Cummings, David S. Brookshire, and William D. Schulze (forthcoming).

REFERENCES

Anderson, Ronald W., "Some Theory of Inverse Demand for Applied Demand Analysis, " European Economic Review, 1.980., 14, 281-98.

Bishop, Richard C. and Heberlein, Thomas A., "Measuring Values of Extra-Market Goods: Are Indirect Measures Biased," American Journal of Agricultural Economics, December 1979, 61, 926-30.

Coursey, Don L., Schulze, William D. and Hovis, John J., "On the Supposed Disparity Between Willingness to Accept and Willingness to Pay Measures of Value: A Comment," mimeo., University of Wyoming, Department of Economics, Laramie, Wyoming, January 1984.

Cummings, Ronald G., Brookshire, David S. and Schulze, William D., Valuing Public Goods: An Assessment of the Contingent Valuation Method, Totowa, N. J.: Rowman and Allanheld, forthcoming.

Diewert, W. E., "A Note on Aggregation and Elasticities of Substitution," Canadian Journal of Economics, February 1974, 7, 12-20.

Freeman, A. Myrick, The Benefits of Environmental Improvement: Theory and Practice, Baltimore: Johns Hopkins University Press, 1979.

Gordon, Irene M. and Knetsch, Jack L., "Consumer's Surplus Measures and the Evaluation of Resources," Land Economics, February 1979, 55, 1-10.

German, W. M., "Tricks With Utility Functions," inM. Artis and R. Nobay, eds., Essays in Economic Analysis, New York: Cambridge University press, 1976.

Hanemann, W. Michael, "Quality and Demand Analysis," in Gordon C. Rausser, cd., New Directions in Econometric Modeling and Forecasting in U. S. Agriculture, Amsterdam: North Holland Publishing Co., 1982.

Just, Richard E., Hueth, Darrell L. and Schmitz, Andrew, Applied Welfare Economics and Public Policy, Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1982.

Knetsch, Jack L. and Sinden, J. A., "Willingness to Pay and Compensation Demanded: Experimental Disparity in Measures of Value," Quarterly Journal of Economics, August 1984, 507-21.

Mäler, Karl-Göran, Environmental Economics: A Theoretical Inquiry, Baltimore: Johns Hopkins University Press, 1974.

Neary, J. P. and Roberts, K. W. S., "The Theory of Household Behavior Under Rationing," European Economic Review, 1980, 13, 25-42.

Randall, Alan and Stoll, John R., "Consumer's Surplus in Commodity Space," American Economic Review, June 1980, 71, 449-57.

Thayer, Mark A., "Contingent Valuation Techniques for Assessing Environmental Impacts: Further Evidence," Journal of Environmental Economics and Management, 1981, 8, 27-44.

Willig, Robert, "consumer's Surplus Without Apology," American Economic Review, September 1976, 66, 589-97.